Math 8 week of 9/30 - 10/4/19

Pulling a One-Eighty!

1

Triangle Sum and Exterior Angle Theorems

WARM UP

Solve each equation for x.

1. $x + 105 = 180$

2. $2x + 65 = 180$

3. $x + (x + 30) + 2x = 180$

4. $(90 - x) + 2x + x = 180$

LEARNING GOALS

- Establish the Triangle Sum Theorem.
- Explore the relationship between the interior angle measures and the side lengths of a triangle.
- Identify the remote interior angles of a triangle.
- Identify the exterior angles of a triangle.
- Use informal arguments to establish facts about exterior angles of triangles.
- Explore the relationship between the exterior angle measures and two remote interior angles of a triangle.
- Prove the Exterior Angle Theorem.

KEY TERMS

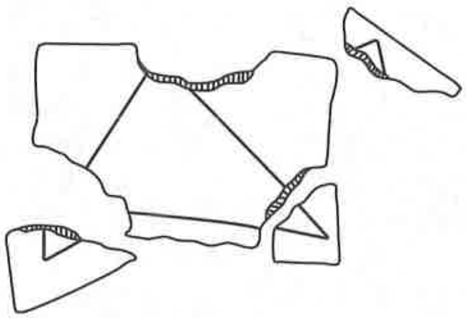
- Triangle Sum Theorem
- exterior angle of a polygon
- remote interior angles of a triangle
- Exterior Angle Theorem

You already know a lot about triangles. In previous grades you classified triangles by side lengths and angle measures. What special relationships exist among the interior angles of a triangle and between interior and exterior angles of a triangle?

Getting Started

Rip 'Em Up

Draw any triangle on a piece of patty paper. Tear off the triangle's three angles. Arrange the angles so that they are adjacent angles.



1. **What do you notice about these angles? Write a conjecture about the sum of the three angles in a triangle.**

2. **Compare your angles and your conjecture with your classmates'. What do you notice?**

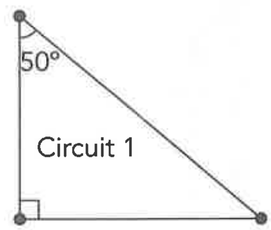
ACTIVITY 1.1

Analyzing Angles and Sides

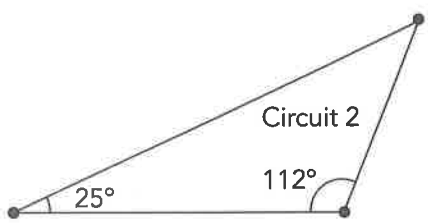
In the previous activity, what you noticed about the relationship between the three angles in a triangle is called *The Triangle Sum Theorem*. The **Triangle Sum Theorem** states that the sum of the measures of the interior angles of a triangle is 180°.

Trevor is organizing a bike race called the Tri-Cities Criterium. Criteriums consist of several laps around a closed circuit. Based on the city map provided to him, Trevor designs three different triangular circuits and presents scale drawings of them to the Tri-Cities Cycling Association for consideration.

1. Classify each circuit according to the type of triangle created.



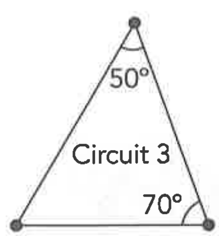
2. Use the Triangle Sum Theorem to determine the measure of the third angle in each triangular circuit. Label the triangles with the unknown angle measures.



3. Measure the length of each side of each triangular circuit. Label the side lengths in the diagram.

The sharper the angles on a race course, the more difficult the course is for cyclists to navigate.

4. Perform the following tasks for each circuit.



a. List the angle measures from least to greatest.

b. List the side lengths from shortest to longest.

c. Describe what you notice about the location of the angle with the least measure and the location of the shortest side.

d. Describe what you notice about the location of the angle with the greatest measure and the location of the longest side.

5. Traci, the president of the Tri-Cities Cycling Association, presents a fourth circuit for consideration. The measures of two of the interior angles of the triangle are 57° and 61°. Determine the measure of the third angle, and then describe the location of each side with respect to the measures of the opposite interior angles without drawing or measuring any part of the triangle.

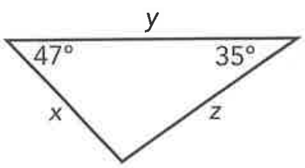
a. measure of the third angle

b. longest side of the triangle

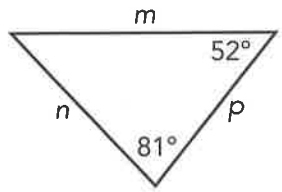
c. shortest side of the triangle

6. List the side lengths from shortest to longest for each diagram.

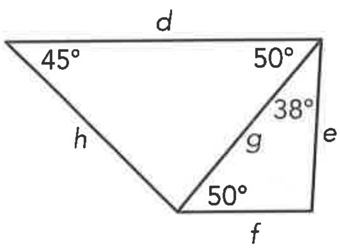
a.

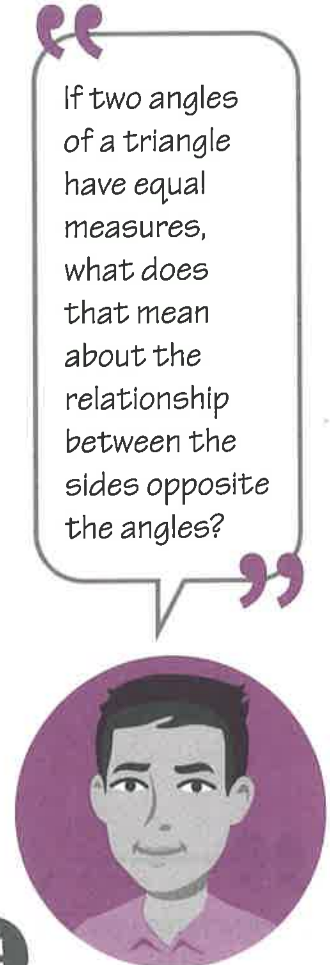


b.



c.



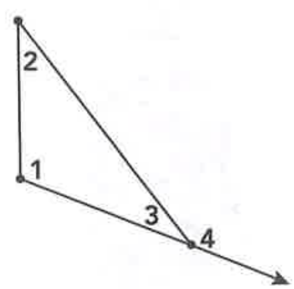


ACTIVITY 1.2

Exterior Angle Theorem

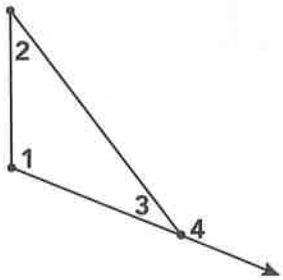
You now know about the relationships among the angles inside a triangle, the *interior angles of a triangle*, but are there special relationships between interior and *exterior angles* of a triangle?

An **exterior angle of a polygon** is an angle between a side of a polygon and the extension of its adjacent side. It is formed by extending a ray from one side of the polygon.



In the diagram, $\angle 1$, $\angle 2$, and $\angle 3$ are the interior angles of the triangle, and $\angle 4$ is an exterior angle of the triangle.

1. **Make a conjecture about the measure of the exterior angle in relation to the measures of the other angles in the diagram.**



2. Let's investigate the relationships among measures of the angles in the diagram.

a. What does $m\angle 1 + m\angle 2 + m\angle 3$ equal? Explain your reasoning.

b. What does $m\angle 3 + m\angle 4$ equal? Explain your reasoning.

c. State a relationship between the measures of $\angle 1$, $\angle 2$, and $\angle 4$. Explain your reasoning.

3. In a triangle, for each exterior angle there are two "remote" interior angles.

a. Why would $\angle 1$ and $\angle 2$ be referred to as "remote" interior angles with respect to the exterior angle, $\angle 4$?

b. Extend another side of the triangle and label the exterior angle $\angle 5$. Then name the two remote interior angles with respect to $\angle 5$.

The **remote interior angles of a triangle** are the two angles that are non-adjacent to the specified exterior angle.

4. Rewrite $m\angle 4 = m\angle 1 + m\angle 2$ using the terms *sum, remote interior angles of a triangle, and exterior angle of a triangle.*

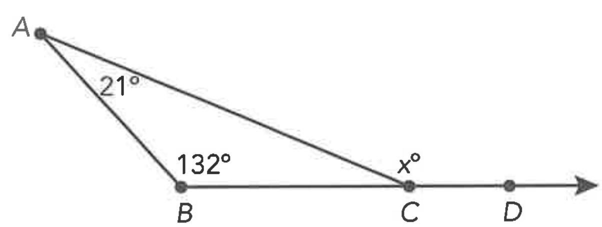
5. The original diagram was drawn as an obtuse triangle with one exterior angle. If the triangle had been drawn as an acute or right triangle, would this have changed the relationship between the measure of the exterior angle and the sum of the measures of the two remote interior angles? Explain your reasoning.

Was your conjecture from Question 1 correct? If so, you have proven an important theorem in the study of geometry!

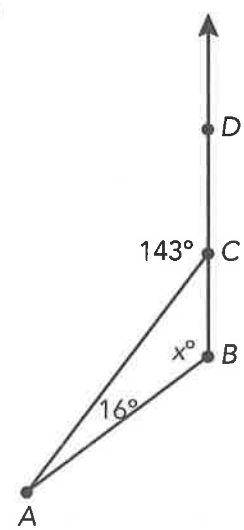
The **Exterior Angle Theorem** states that the measure of the exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles of the triangle.

6. Use the Exterior Angle Theorem to determine each unknown angle measure.

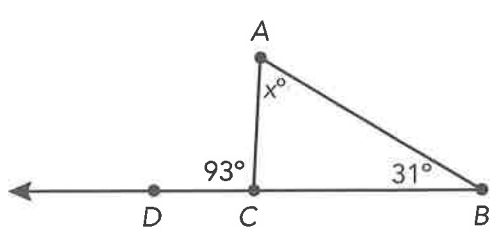
a.



b.

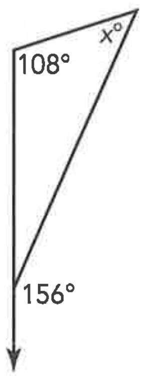


c.

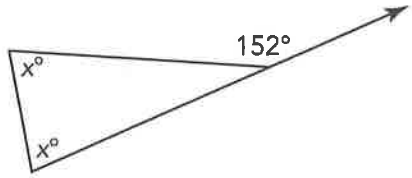


7. **Write and solve an equation to determine the value of *x* in each diagram.**

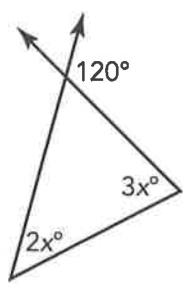
a.



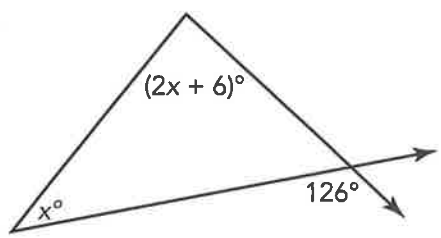
b.



c.



d.

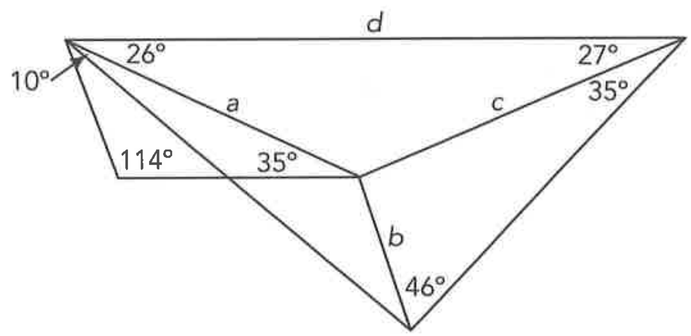


NOTES

TALK the TALK

So Many Angles!

1. Consider the diagram shown.

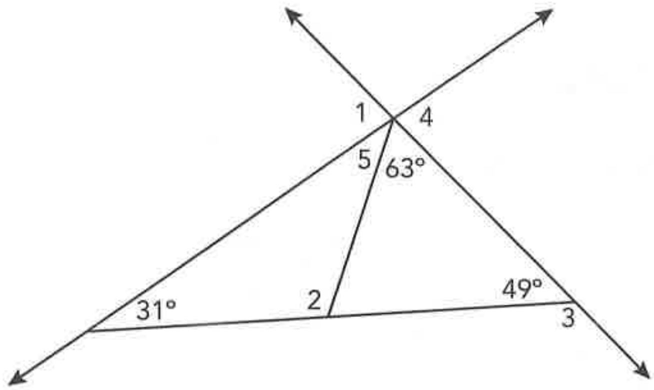


a. Determine the measures of the eight unknown angle measures inside the figure.

b. List the labeled side lengths in order from least to greatest.

NOTES

2. Determine the unknown angle measures in the figure.



Assignment

hw math 8
9/30- 10/4

Write

Write the term that best completes each statement.

1. The ______________________________
   states that the sum of the measures of the interior angles of a triangle is 180°.
2. The ______________________________
   states that the measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles of the triangle.
3. The ______________________________
   are the two angles that are non-adjacent to the specified exterior angle.
4. A(n) ______________________________ is
   formed by extending a side of a polygon.

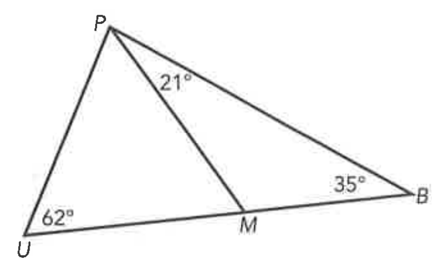
Remember

The sum of the measures of the interior angles of a triangle is 180°.

The measure of the exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles of the triangle.

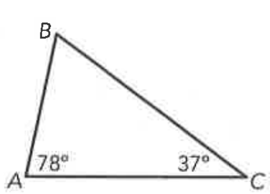
Practice

1. Use the figure shown to answer each question.

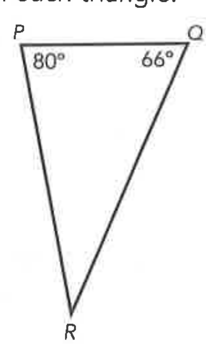


   a. Explain how you can use the Exterior Angle Theorem to calculate the measure of ∠*PMU*.
   b. Calculate the measure of ∠*PMU*.
   c. Explain how you can use the Triangle Sum Theorem to calculate the measure of ∠*UPM*.
   d. Calculate the measure of ∠*UPM*.
   e. List the sides of Δ*PMB* in order from shortest to longest. Explain how you determined your answer.
   f. List the sides of Δ*PUB* in order from shortest to longest. Explain how you determined your answer.
2. Determine the measure of the unknown angle in each triangle.

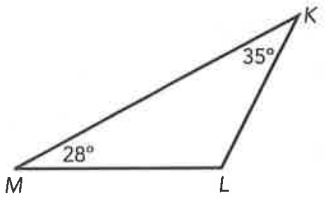
   a.



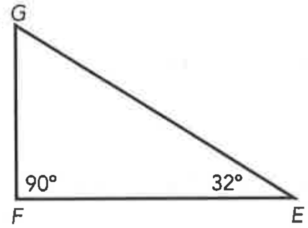
   b.



c.

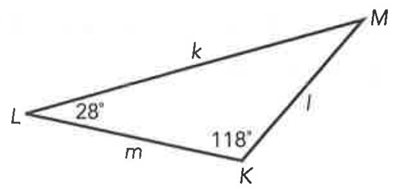


d.

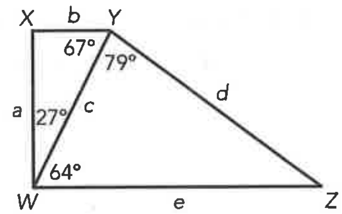


3. List the side lengths from shortest to longest for each diagram.

a.

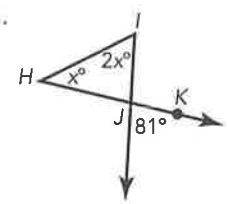


b.

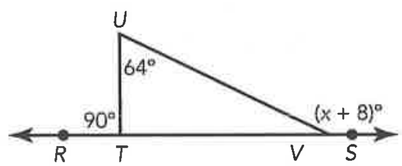


4. Determine the value of x in each diagram.

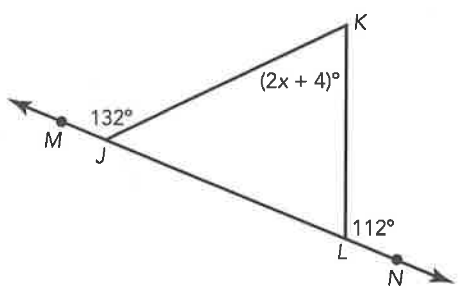
a.



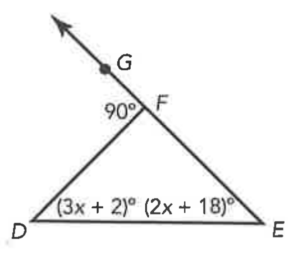
b.



c.



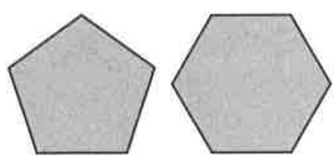
d.



Stretch

To tessellate a plane means to cover a surface by repeated use of a single shape or design without gaps or overlaps. M.C. Escher was a Dutch graphic artist who is famous for his tessellations, perspective drawings, and impossible spaces.

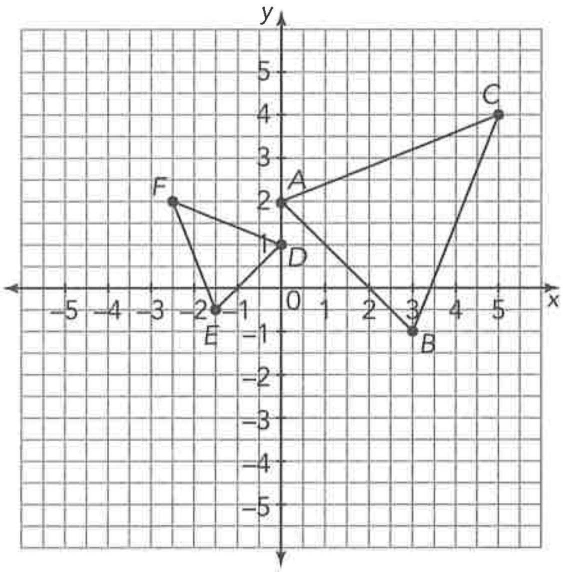
Not all shapes or patterns can be tessellated. Use what you know about interior and exterior angles to show why it is possible to tessellate with a regular hexagon but not with a regular pentagon.



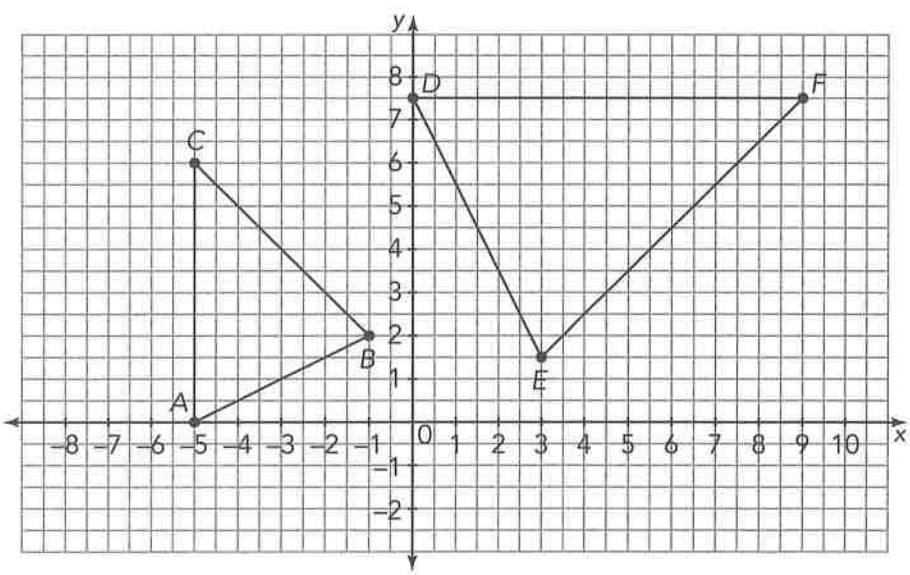
Review

1. Triangle *ABC* is similar to Triangle *DEF*. Determine a sequence of transformations that maps ΔABC onto ΔDEF.

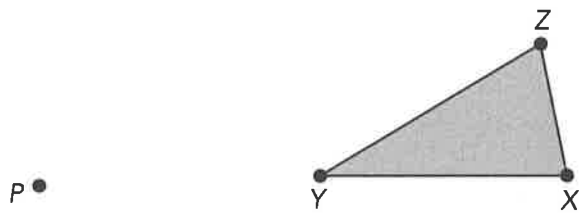
a.



b.

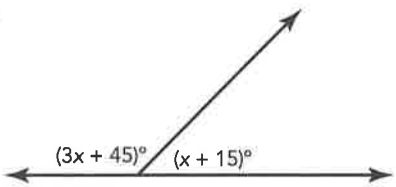


2. Dilate ΔXYZ by the given scale factor, using point P as the center of dilation.
   a. Dilate by a scale factor of $\frac{3}{4}$.
   b. Dilate by a scale factor of 1.5.



3. Calculate the measure of each angle.

a.



b.