

What's the Point?

Math 8
Week of
2/10 - 2/14/
20

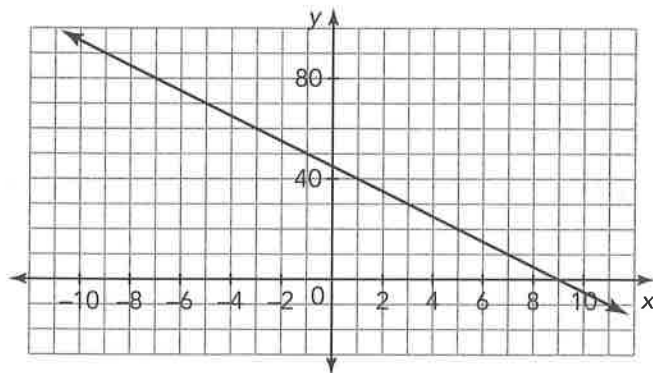
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Point-Slope Form of a Line

WARM UP

Write an equation for each linear relationship.

1. The contestant at a game show had already won a total of \$2750 when the game show was continued today. He earns an additional \$250 for each question he answers correctly today.
- 2.



LEARNING GOALS

- Use the slope formula to derive the point-slope form of a linear equation.
- Construct an equation in point-slope form to model a linear relationship between two quantities.
- Write equations for vertical and horizontal lines.

KEY TERM

- point-slope form

You have used the slope-intercept form to represent linear relationships. Are there other forms of a linear equation that you can use? How do you write equations for horizontal and vertical lines?

Getting Started

Draining the Pool

Cyrus and Ava are pool cleaners who have been hired to drain the community diving pools at the end of the summer. They are comparing the rate at which the two pools drain.

1. For each pool, write an equation to represent the linear relationship.
 - a. Cityscape Diving Pool is at a water level of 14 feet and drains at a rate of 3 feet per hour.

I wonder if there is a way to make writing the equation of a line more efficient.



- b. Bayside Diving Pool is at a water level of 15 feet after draining for 2 hours and at 12 feet after draining for 4 hours.

2. Compare your process for writing each equation. How are the processes different?

Writing Equations in Point-Slope Form



In the previous lesson, you used the slope, the y -intercept, and the slope formula to write a linear equation. You can also determine the equation of a line without knowing the y -intercept.

WORKED EXAMPLE

To write an equation of a line from a table of values, you can use the slope formula.

- First, calculate the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 5}{2 - 4}$$

$$= \frac{1}{-2} = -\frac{1}{2}$$

- Next, choose any point from the table.

$$(2, 6)$$

- Then, substitute what you know into the slope formula: $m = -\frac{1}{2}$, $(2, 6)$, and the unknown point (x, y) .

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-\frac{1}{2} = \frac{y - 6}{x - 2}$$

- Finally, rewrite the equation with no variables in a denominator.

$$-\frac{1}{2} = \frac{y - 6}{x - 2}$$

$$-\frac{1}{2}(x - 2) = y - 6$$

The equation is $y - 6 = -\frac{1}{2}(x - 2)$.

x	y
2	6
4	5
6	4

This linear equation in the worked example is written in *point-slope form*. The **point-slope form** of a linear equation is $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) is any point on the line.

1. Solve the equation in the worked example for y so that the linear equation is in slope-intercept form. What unique information does each form of the linear equation provide? How are they similar?

Write the equation for each linear relationship in point-slope form.

2. The slope is -8 . The point $(3, 12)$ lies on the line.

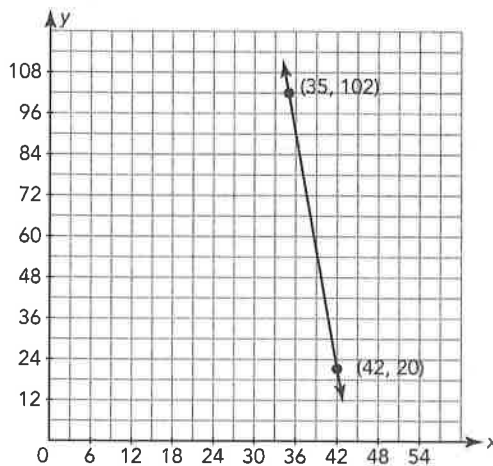
3. $(429, 956)$ and $(249, 836)$

Use the given information to write an equation to represent each linear relationship in either slope-intercept form or in point-slope form. Describe your process.

4. The cost to ship a package in the mail includes a basic shipping charge plus an additional cost per number of pounds the package weighs. A three-pound package costs \$6.30 to ship. A ten-pound package costs \$14 to ship.

5. $m = -\frac{3}{8}$; $(50, 7)$

6.



7. $(7, 15)$ and $(-39, -8)$

8.

x	y
-5	-6
1	-6
2	-6

9. Examine each detail about a linear relationship that you may be provided. Which form of the equation do you prefer to use in each case? Explain your reasoning.

a. slope and y -intercept

b. two points

c. slope and a point other than the y -intercept



Horizontal and vertical lines represent linear relationships, but their equations are different from the equations of lines that are not horizontal or vertical.

x	y
-5	-6
1	-6
2	-6



1. Consider the equation, $y = -6$, that you wrote for the table shown in the previous activity.

a. How is this equation different from the other equations? What is its slope?

b. Describe the graph of the coordinate pairs in this table. Why does the value of its slope make sense?

c. Explain why the equation makes sense in terms of the graph and the table.

2. Write an equation for each linear relationship. Describe the graph of the linear relationship. State the slope and y-intercept.

a.

x	y
-7	11
-2	11
0	11

b. A line that passes through $(-15, -3.75)$ and $(89, -3.75)$

3. Consider a new table of values representing a linear relationship.

a. Explain how this table is similar to and different from the tables in Questions 1 and 2.

b. Write an equation for the linear relationship in the table.

c. Describe the graph of this linear relationship.

d. Use the slope formula to calculate the slope between two points in the table. What do you notice?

e. What is the y -intercept of this linear relationship? Explain why this makes sense.

x	y
-2	5
-2	14
-2	29

4. Write an equation for each linear relationship. Describe the graph of the linear relationship.

a.

x	y
$\frac{17}{2}$	-18
$\frac{17}{2}$	23
$\frac{17}{2}$	267

b. A line that passes through $(-7, -973)$ and $(-7, 542)$

c. Create an additional table of values and write the equation for a vertical line.

In a horizontal line there is no change in the y -values as the x -values change. Therefore, the slope is 0. A horizontal line has zero steepness. In a vertical line there is no change in the x -values as the y -values change. Therefore, the slope is undefined. A vertical line has an undefined steepness.



1. Carefully cut out the graphs, tables, contexts, and equations located the end of the lesson. Match each equation with its correct graph, table, or context. Explain how you matched the equations with the representations.

2. Compare the graphs.

- a. How are they different? How are these differences reflected in the slope-intercept form of their equation?

- b. Identify the y-intercept for each graph. How can you determine this point in the slope-intercept form of the equation for each graph?

- c. Identify the slope for each graph. How is the slope represented in the slope-intercept form of each equation?

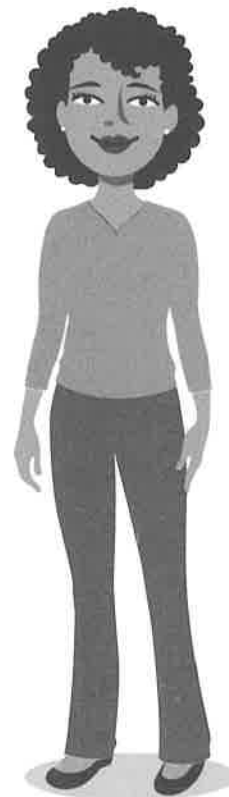
3. Analyze the equation for each table.

a. Determine the coefficient of x for each linear relationship using the slope formula.

b. How can the number that is added in each equation written in slope-intercept form be determined from the table?

4. Analyze the equation for each context. Explain what each term of the equation means in each context.

“
Can you remember the ways to determine the rate of change from a table?
”



TALK the TALK**Say What?**

You have learned about two forms of a linear equation: the slope-intercept form, $y = mx + b$, and the point-slope form, $y - y_1 = m(x - x_1)$.

1. What information can you determine about each line by looking at the structure of the equation?

a. $y = \frac{3}{5}x - 4$

b. $y - 6 = 2(x + 1)$

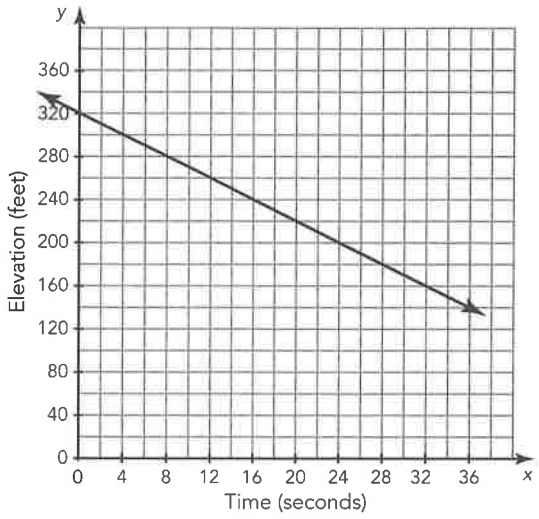
c. $y + 4 = 2x$

d. $y = -\frac{2}{7}x$

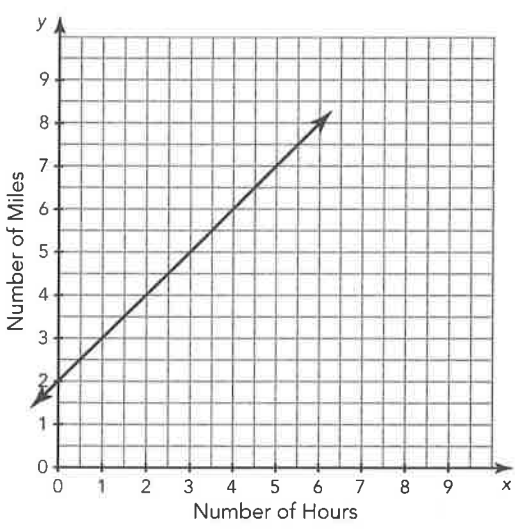
e. $y + 5 = -(x - 4)$

f. $y = 19$

2. Create a context that represents a linear relationship that passes through the point $(2, 56)$ and has an increasing slope. Then write the equation of the line in point-slope form and slope-intercept form.

**A****B**

Michele read the first 40 pages of a mystery novel before she fell asleep. The next day, she read one page every two minutes until she finished the book, which was a total of 325 pages.

C**D**

Time (hours)	Water Level (feet)
x	y
2	15
4	13.5
8	10.5
10	9

E

Number of Games Ron Won Today	Number of Credits on Ron's Player's Card
x	y
12	216
18	264
25	320
40	440

F

Bella's Pizza Shop charges \$4.50 for a small pizza, \$7.00 for a medium pizza, and \$9.00 for a large pizza. Additional toppings cost extra depending on the size of the pizza ordered. Bruce ordered a large pizza with three toppings that cost a total of \$12.60.





$$y = 1.2x + 9$$

$$y = -\frac{3}{4}x + \frac{33}{2}$$

$$y = \frac{1}{2}x + 40$$

$$y - x = 2$$

$$y - 200 = -5(x - 24)$$

$$y = 8x + 120$$



Assignment

Write

Compare the slope-intercept and point-slope forms of a linear equation.

Remember

The point-slope form of a linear equation is $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) is a point on the line. The slope of a horizontal line is 0. The slope of a vertical line is undefined.

Practice

Write an equation in point-slope form.

- $m = 2$; $(5, 6)$
- $m = -9.2$; $(-17, 10)$
- $(-2, -3)$ and $(8, -8)$
- $(79, 52)$ and $(-87, 550)$
- A photography studio charges \$50 for a sitting fee and 6 prints. Luigi increased his order to 11 prints and paid \$65.
- Zellie is taking the stairs in her building from her floor to the top of the building. After 2 minutes, she was 100 steps from the bottom floor. After 5 minutes, she was 196 steps from the bottom floor.

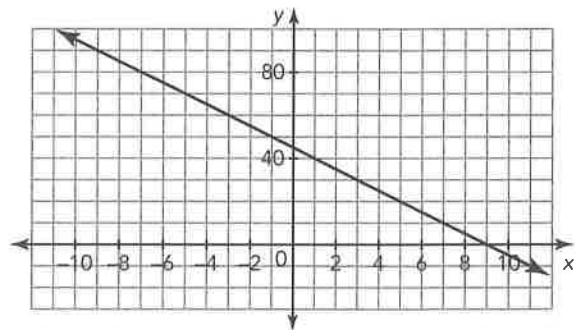
Write an equation in any form.

- A newspaper charges a flat fee plus a charge per day to place a classified ad.

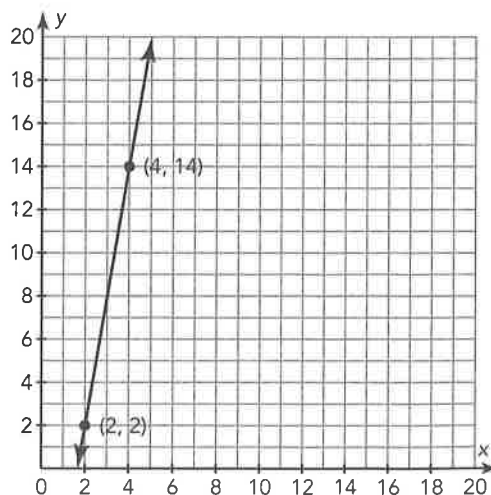
Number of Days	Total Charge (\$)
2	8.00
4	13.00
6	18.00

- | x | y |
|-----|-----|
| -10 | 50 |
| -2 | 10 |
| 4 | -20 |
| 14 | -70 |

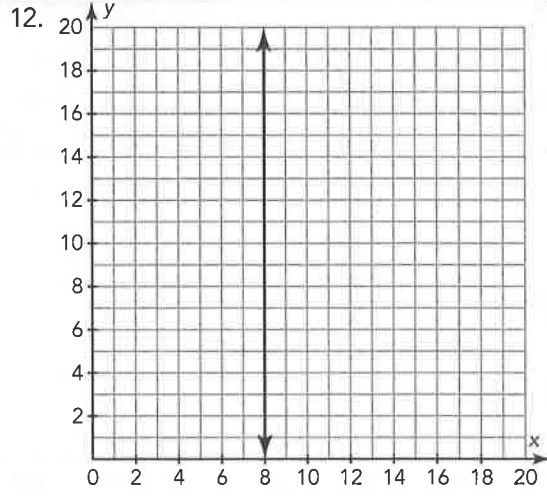
8.



10.



11. Pedro is traveling on a toll road. He plans to exit the road 5 miles ahead at First Avenue and pay \$1.75. He changes his plans and travels 9 miles to Butler Street and pays \$2.75.



Stretch

To convert an equation from point-slope to slope-intercept form, you can solve the equation for y . How do you convert from slope-intercept to point-slope form? Rewrite each equation in point-slope form using only algebraic properties. What is special about the ordered pair now visible in the equation?

- $y = 2x - 7$
- $y = -5x + 15$

Review

- Write an equation in slope-intercept form with the given characteristics.
 - The line is increasing and passes through the point $(0, -10)$. The slope of the line is less steep than the slope of the line represented by the equation $y = x + 8$.
 - The line is decreasing and passes through the point $(0, 5)$. The slope of the line is more steep than the slope of the line represented by the equation $y = -\frac{1}{4}x - 4$.
- For the linear equation $x = 4y - 5$, complete each task.
 - Use a table of values to graph the linear equation.
 - Use the points on the graph to sketch similar triangles that may be used to show that the slope of a non-vertical line is the same between any two points on the line.
 - Verify that the slopes are the same.
- Solve each problem.
 - What is a 15% tip for a restaurant bill of \$24?
 - A \$50 item was marked up 20%. What is the total increased cost of the item?