

Math 7 week of
11/27/20

1

Equal Groups

• Multiplying and Dividing Integers

WARM UP

Determine each sum.

1. $-2\frac{3}{4} + (-2\frac{3}{4})$
2. $-9.502 - 4.239$
3. $-3 + 8 + (-2)$
4. $5 - 16 + 7 + (-1)$

LEARNING GOALS

- Multiply integers using models.
- Develop rule for multiplying integers.
- Develop rule for dividing integers.

You know the reasoning and rules to add and subtract integers. How do you multiply and divide integers?

Getting Started

Addition or Multiplication?

Consider the addition problem $(-8) + (-8) + (-8) + (-8)$.

1. Rewrite the addition problem as a multiplication problem.

2. Is the product from Question 1 positive or negative?
Explain your reasoning.

Consider the addition problem $(-1) + (-1) + (-1) + (-1) + (-1)$.

3. Rewrite the addition problem as a multiplication problem.

4. Is the product from Question 3 positive or negative?
Explain your reasoning.

5. What relationship helped you answer Questions 2 and 4?

Modeling the Multiplication of Integers



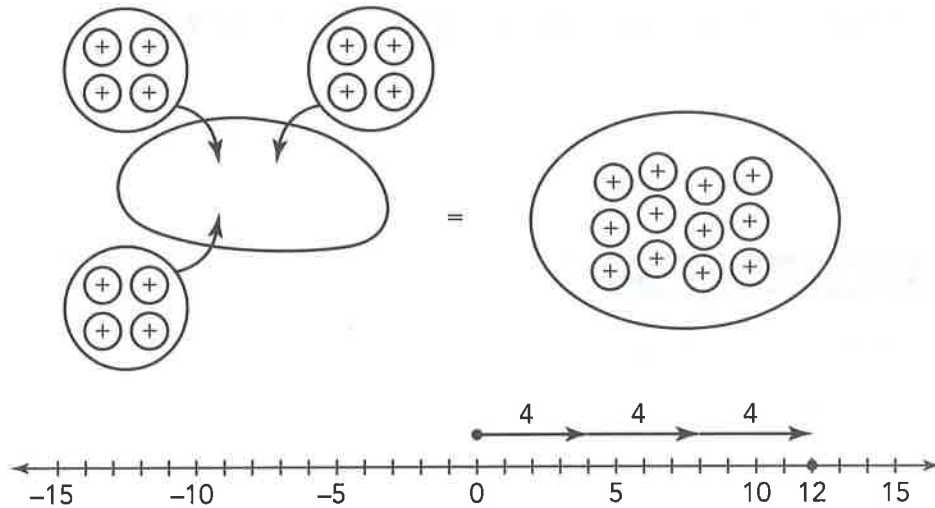
When thinking about multiplying integers, remember that multiplication can be represented as repeated addition.

WORKED EXAMPLE 1

Consider the expression 3×4 .

As repeated addition, it is represented as $4 + 4 + 4$.

You can think of 3×4 as three groups of 4.

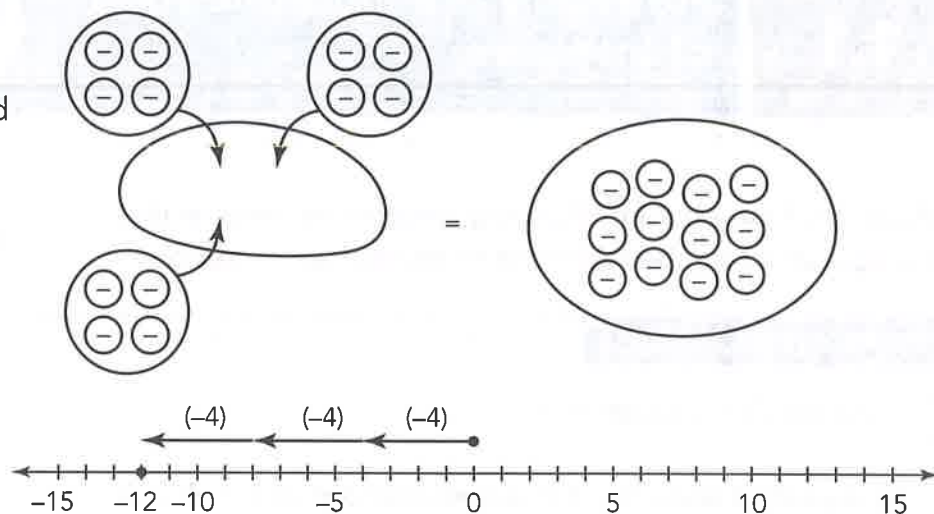


1. Explain how the number line in Worked Example 1 illustrates 3×4 .

WORKED EXAMPLE 2

Consider the expression $3 \times (-4)$. As repeated addition, it is represented as $(-4) + (-4) + (-4)$.

You can think of $3 \times (-4)$ as three groups of (-4) .



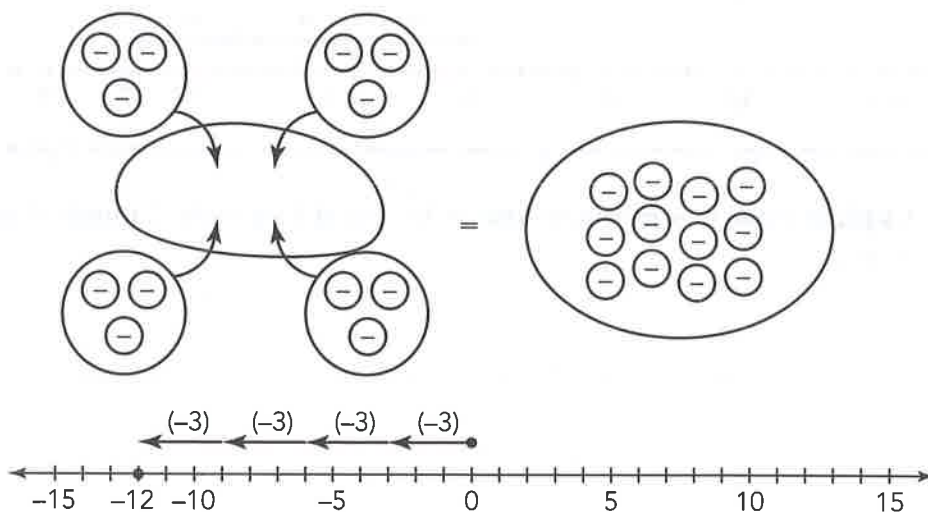
2. Explain how each model in Worked Example 2 can be interpreted as three groups of the opposite of 4.

How is the number line model similar to models you have used to add positive numbers?



WORKED EXAMPLE 3

Consider the expression $4 \times (-3)$.

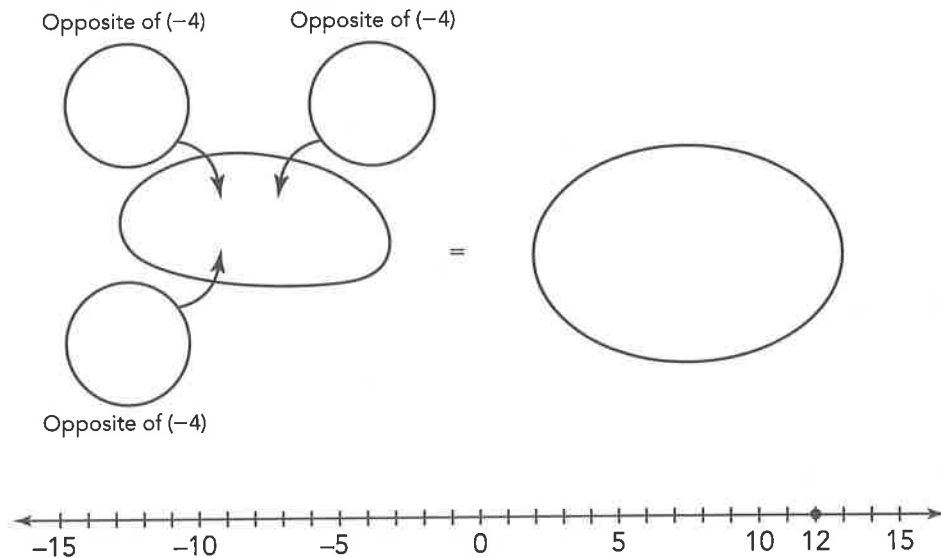


3. Interpret each model in Worked Example 3.

WORKED EXAMPLE 4

Consider the expression $(-3) \times (-4)$.

You know that $3 \times (-4)$ means "three groups of (-4) " and that -3 means "the opposite of 3." So, $(-3) \times (-4)$ means "the opposite of 3 groups of (-4) ."



Think about how the 4 worked examples are alike and how they are different.

4. Complete the model by drawing in the appropriate counters and the appropriate rays in Worked Example 4.

- Determine the product and explain how your models illustrate this product.
- This expression can be written as $-((-4) + (-4) + (-4))$. Rewrite the expression as the opposite of a number: $-(\text{_____})$.
- How does the expression you wrote in part (b) relate to the product? Explain why this makes sense.

Use the examples if you need help.

5. Draw either a number line or two-color counter model to determine each product. Describe the expression in words.

a. 2×3

b. $2 \times (-3)$

c. $(-2) \times 3$

d. $(-2) \times (-3)$

6. Complete the table.

Expression	Description	Addition Sentence	Product
3×5	Three groups of 5	$5 + 5 + 5 = 15$	15
$(-3) \times 5$			
$3 \times (-5)$			
$(-3) \times (-5)$			

7. What do you notice about the products and their signs across the problems in this activity?

Signed Multiplication Facts



Analyze the sequence of products with 4.

$$4 \times 5 = 20$$

$$4 \times 4 = 16$$

$$4 \times 3 = 12$$

$$4 \times 2 = 8$$

$$4 \times 1 = 4$$

$$4 \times 0 = 0$$

1. What pattern do you notice in the products as the numbers multiplied by 4 decrease?

2. Continue the pattern to determine each product.

a. $4 \times (-1) =$ _____

b. $4 \times (-2) =$ _____

c. $4 \times (-3) =$ _____

3. Describe the pattern(s) that you notice in the new products.

Analyze the sequence of products with -5 .

$$-5 \times 5 = -25$$

$$-5 \times 4 = -20$$

$$-5 \times 3 = -15$$

$$-5 \times 2 = -10$$

$$-5 \times 1 = -5$$

$$-5 \times 0 = 0$$

4. Describe the pattern and then extend it by writing the next three number sentences.

5. How do these products change as the numbers multiplied by -5 decrease?

Look back at the products you have determined in this lesson to answer each question.

6. Describe the sign of the product of two integers when:

a. they are both positive. b. they are both negative.

c. one is positive and one is negative. d. one is zero.

7. If you know that the product of two integers is negative, what can you say about the two integers? Give examples.

“Does the order in which you multiply the integers matter?”

8. Describe a rule that will help you multiply any two integers.

9. Use your rule to evaluate each expression.

a. $6 \times 5 = \underline{\hspace{2cm}}$ b. $-8 \times 7 = \underline{\hspace{2cm}}$
 $6 \times (-5) = \underline{\hspace{2cm}}$ $-8 \times (-7) = \underline{\hspace{2cm}}$
 $-6 \times 5 = \underline{\hspace{2cm}}$ $8 \times (-7) = \underline{\hspace{2cm}}$
 $-6 \times (-5) = \underline{\hspace{2cm}}$ $8 \times 7 = \underline{\hspace{2cm}}$

c. $-3 \times 2 \times (-4) = \underline{\hspace{2cm}}$
 $-3 \times (-2) \times (-4) = \underline{\hspace{2cm}}$
 $3 \times (-2) \times 4 = \underline{\hspace{2cm}}$
 $-3 \times (-2) \times 4 = \underline{\hspace{2cm}}$
 $3 \times 2 \times (-4) = \underline{\hspace{2cm}}$
 $-3 \times 2 \times 4 = \underline{\hspace{2cm}}$



10. Describe the sign of each product and how you know.

a. the product of three negative integers

b. the product of four negative integers

c. the product of seven negative integers

d. the product of ten negative integers

11. What is the sign of the product of any odd number of negative integers? Explain your reasoning.

12. What is the sign of the product of three positive integers and five negative integers? Explain your reasoning.

Create some examples to test if you are not sure how to answer these questions.

“
How is determining the sign of the product different from when you add and subtract signed numbers?
”





When you studied division in elementary school, you learned that multiplication and division were inverse operations. For every multiplication fact, you can write a corresponding division fact.

WORKED EXAMPLE

Consider the fact family for 4, 5, and 20.

$$4 \times 5 = 20$$

$$5 \times 4 = 20$$

$$20 \div 4 = 5$$

$$20 \div 5 = 4$$

Similarly, you can write fact families for integer multiplication and division.

EXAMPLES:

$$-7 \times 3 = -21$$

$$-8 \times (-4) = 32$$

$$3 \times (-7) = -21$$

$$-4 \times (-8) = 32$$

$$-21 \div (-7) = 3$$

$$32 \div (-8) = -4$$

$$-21 \div 3 = -7$$

$$32 \div (-4) = -8$$

1. What pattern(s) do you notice in each fact family?

2. Write a fact family for -6 , 8 , and -48 .

3. Fill in the unknown numbers to make each number sentence true.

a. $56 \div (-8) = \underline{\hspace{2cm}}$

b. $28 \div (-4) = \underline{\hspace{2cm}}$

c. $-63 \div \underline{\hspace{2cm}} = -7$

d. $24 \div \underline{\hspace{2cm}} = -8$

e. $\underline{\hspace{2cm}} \div (-8) = -4$

f. $-105 \div \underline{\hspace{2cm}} = -5$

g. $\underline{\hspace{2cm}} \div (-8) = 0$

h. $-26 \div \underline{\hspace{2cm}} = -1$

4. Describe the sign of the quotient of two integers when:

a. both integers are positive.

b. one integer is positive and one integer is negative.

c. both integers are negative.

d. the dividend is zero.

5. How do the answers to Question 4 compare to the answers to the same questions about the multiplication of two integers? Explain your reasoning.

Use fact families to help you determine each answer.

“Remember that a quotient is the answer to a division problem.”



TALK the TALK **What's Your Sign?**

Think about patterns in the signs of sums, differences, products, and quotients of integers.

1. Determine two different sets of single-digit integers that make each number sentence true.

a. $\underline{\quad} \times \underline{\quad} = -42$

b. $\underline{\quad} \times \underline{\quad} = 56$

c. $\underline{\quad} \times \underline{\quad} = 63$

d. $\underline{\quad} \times \underline{\quad} = -48$

2. Complete the table by writing the sign (+, -, or +/-) to describe each sum, difference, product, or quotient.

Description of Integers	Addition (Sum)	Subtraction (Difference)	Multiplication (Product)	Division (Quotient)
two positive integers				
two negative integers				
one positive and one negative integer				

3. Create a true multiplication or division number sentence that meets the given condition.

a. positive product

b. negative product

c. positive quotient

d. negative quotient

Assignment

Write

Explain how you determine the sign of the product or quotient of three of integers.

Remember

To multiply and divide integers, perform the usual multiplication and division algorithms and then apply the correct sign to the product or quotient.

Practice

Draw a two-color counter model to determine each product. Describe the expression in words.

- $6 \times (-3)$
- -2×5
- $-4 \times (-2)$

Complete a number line model to determine each product.

- -2×7
- $-5 \times (-3)$
- $3 \times (-3)$

Determine each product.

- | | |
|-------------------|------------------------------|
| 7. 5×11 | 8. $3 \times 4 \times 5$ |
| $5 \times (-11)$ | $-3 \times (-4) \times 5$ |
| -5×11 | $-3 \times 4 \times 5$ |
| $-5 \times (-11)$ | $-3 \times (-4) \times (-5)$ |
| | $3 \times 4 \times (-5)$ |
| | $3 \times (-4) \times (-5)$ |

Determine the integer that makes each number sentence true.

- | | |
|------------------------------|------------------------------|
| 9. _____ $\times (-9) = -36$ | 10. $3 \times$ _____ $= -24$ |
| 11. $14 \times$ _____ $= 56$ | 12. _____ $\times (-6) = 30$ |
| 13. $9 = (-63) \div$ _____ | 14. $-40 \div$ _____ $= -8$ |
| 15. $16 \div$ _____ $= -8$ | 16. _____ $\div (-6) = -4$ |

Stretch

Multiplication can be represented as repeated addition. Repeated multiplication leads to exponents. Use what you know about multiplying signed numbers to evaluate each expression.

1. $(-3)^3$

2. $(-4)^2$

3. $(-2)^5$

4. $\left(-\frac{1}{2}\right)^2$

What do you notice?

Review

1. The Baby Shop sells baby supplies for new families. They offer different brands of the same items. James and his mom are shopping for his new baby brother. It is James' job to make sure that his mom is making wise purchases. Their first item to purchase is diapers. There are 3 different options for newborn-sized diapers.

Stay-Dry: 108 diapers for \$25.18

UberSoft: 180 diapers for \$39.14

Cuddlies: 160 diapers for \$38.77

- What is a unit rate for the Stay-Dry diapers?
 - What is a unit rate for the UberSoft diapers?
 - What is a unit rate for the Cuddlies diapers?
 - Which kind of diapers should James advise his mom to purchase?
2. Calculate each sum.
- $2\frac{1}{2} + \left(-3\frac{3}{4}\right) + 5\frac{2}{5}$
 - $5\frac{1}{3} + \left(-4\frac{1}{6}\right) + \left(-2\frac{1}{2}\right)$
3. Determine each unit rate.
- $1\frac{1}{4}$ teaspoons baking powder per $\frac{3}{8}$ cup flour
 - $2\frac{2}{5}$ parts ammonia per $1\frac{1}{3}$ parts vinegar